Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Real Analysis

Subject Code: 5SC02REA1 **Branch: M.Sc.(Mathematics)**

Semester: 2 **Date: 25/04/2019** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a) Define: $G_\delta -$set. (02)

b) Define: Measurable function (02)

c) If $A, B \subseteq R$ be such that $m^*(B) = 0$ then prove that $m^*(A \cup B) = m^*(A)$. (02)

d) If A is cantor set then find $m^*(A)$. (01)

Q-2 **Attempt all questions** **(14)**

a) Prove that m is finitely additive and countably subadditive if m is countably subadditive. (07)

b) Prove that outer measure of an interval is its length. (07)

OR

Q-2 **Attempt all questions** **(14)**

a) Prove that P is non-measurable set. Where P contains one element from each equivalence classes E_λ and $\bigcup E_\lambda = X = [0,1)$. (08)

b) Let m be the set of all measurable subsets of R then prove that m is an $\sigma -$ algebra on R. (06)

Q-3 **Attempt all questions** **(14)**

a) If $E_1, E_2, ..., E_n$ be a finite sequence of measurable sets and they are mutually disjoint then for any $A \subseteq R$, $m^*\left(A \cap \left(\bigcup_{i=1}^{n} E_i\right)\right) = \sum_{i=1}^{n} m^*(A \cap E_i)$. (05)

b) Let f be a bounded function on a measurable set E with $m(E)<\infty$ then prove that $\inf\limits_{\psi\ge f}\int\limits_E \psi = \sup\limits_{\phi\le f}\int\limits_E \phi$, for all simple functions ϕ and ψ on E if f is a measurable functions on E. (05)

c) Let $E\subseteq R$ and χ_E is the characteristic function of E on R then χ_E is a measurable function on R iff $E\in m$. (04)

OR

Q-3 Attempt all questions (14)

a) Let f be a bounded function on a measurable set E with $m(E)<\infty$. if $\inf\limits_{\psi\ge f}\int\limits_E \psi = \sup\limits_{\phi\le f}\int\limits_E \phi$, for all simple functions ϕ and ψ on E then prove that f is a measurable functions on E. (05)

b) Let f, g be two measurable function on E, where $E\in$ m then for any $f\cdot g$ are also measurable function on E. (05)

c) Give an example of measurable map which is not a Riemann integrable map and explain. (04)

SECTION – II

Q-4 Attempt the Following questions (07)

a) Explain monotone convergence theorem is false for decreasing function. (02)

b) Check whether or not the function $f(x)=x^2\sin\left(\frac{1}{x}\right)$, if $x\neq 0$, $f(0)=0$ is of bounded variation on $[0,1]$ (02)

c) Define: Positive and negative variation. (02)

d) True/False: $f^-(x)=-\min(f(x),0)$ (01)

Q-5 Attempt all questions (14)

a) State and prove Bounded convergence theorem. (06)

b) Let f, g be two integrable map relative to positive and negative part over E then cf is also an integrable map over E and $\int\limits_E cf = c\int\limits_E f$. (04)

c) State and prove Chebychev's inequality. (04)

OR

Q-5 Attempt all questions (14)

a) State and prove Lebesgue dominated convergence theorem. (07)

b) Define convergence in the sense of measure and also prove that there is a subsequence $\{f_{n_k}\}$ of $\{f_n\}$ pointwise converges to f a.e. on E. (04)

c) State Littlewoods's three principles. (03)

Q-6 **Attempt all questions** **(14)**

a) State and prove Fundamental theorem of integral calculus. (07)

b) State and prove Jordan's lemma. (05)

c) Write monotone convergence theorem. (02)

OR

Q-6 **Attempt all Questions** **(14)**

a) F is absolutely continuous function on $[a,b]$ iff F is indefinite integral. (07)

b) Define: Bounded variation and if $f, g \in BV[a,b]$ then prove that $fg \in BV[a,b]$. (05)

c) State Beppo-Levi's theorem. (02)